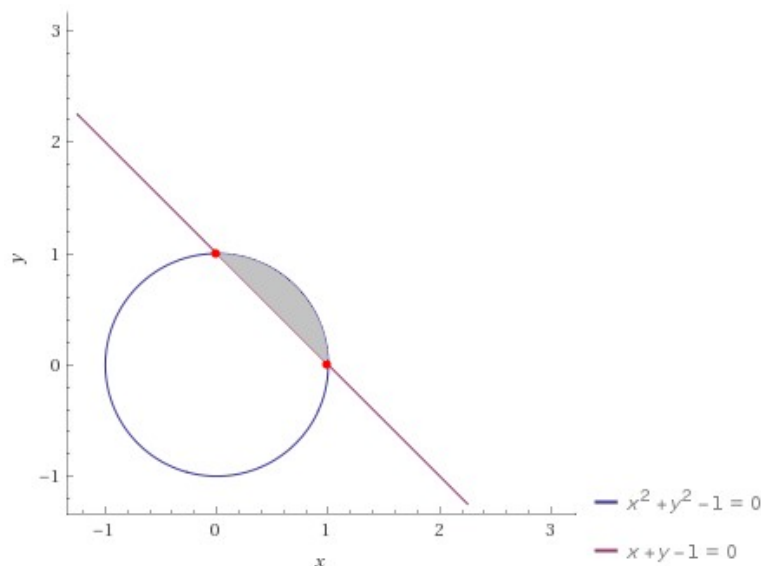


UKÁZKOVÉ PŘÍKLADY NA DVOJNÝ INTEGRÁL

Cvičení 1. Spočítejte $\iint_B xy \, dx \, dy$, kde $B = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 - 1 \leq 0, x + y - 1 \geq 0\}$.

Obor integrace:

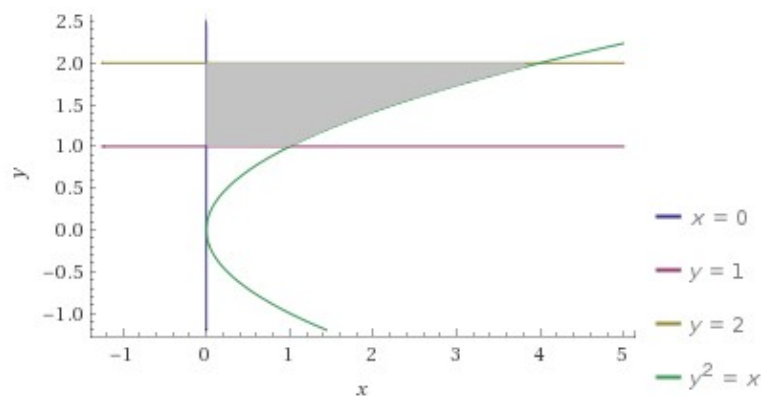


Řešení:

$$\begin{aligned} \iint_B xy \, dx \, dy &= \int_0^1 \int_{1-x}^{\sqrt{1-x^2}} xy \, dy \, dx = \int_0^1 x \left[\frac{y^2}{2} \right]_{y=1-x}^{\sqrt{1-x^2}} dx = \int_0^1 x \left(\frac{1-x^2}{2} - \frac{(1-x)^2}{2} \right) dx \\ &= \int_0^1 x^2 - x^3 \, dx = \frac{1}{12} \end{aligned}$$

Cvičení 2. Spočítejte $\iint_B e^{\frac{x}{y}} \, dx \, dy$, kde $B = \{(x, y) \in \mathbb{R}^2 : x \leq 0, y \geq 1, y \leq 2, y^2 \geq x\}$.

Obor integrace:

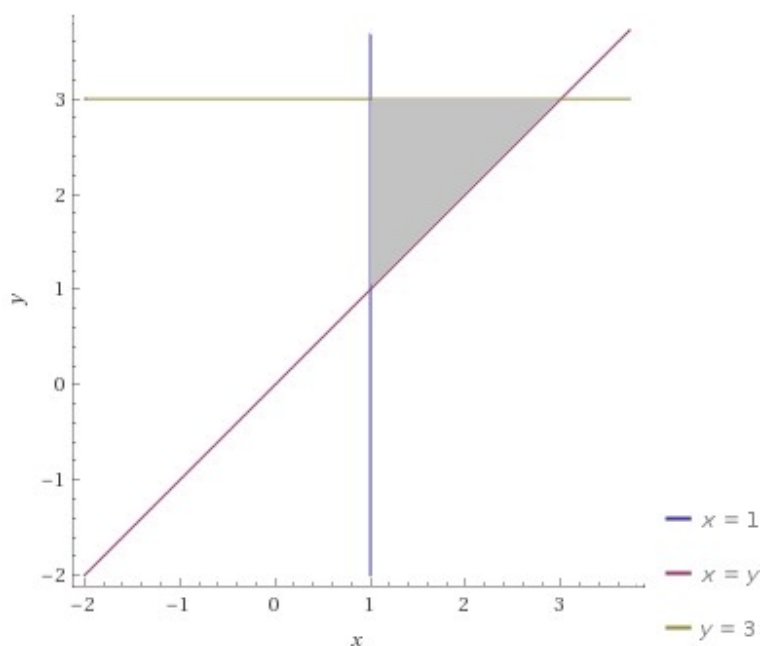


Řešení:

$$\iint_B xy \, dx \, dy = \int_1^2 \int_0^{y^2} e^{\frac{x}{y}} \, dx \, dy = \int_1^2 \left[ye^{\frac{x}{y}} \right]_{x=0}^{y^2} dy = \int_1^2 ye^y - y \, dy = \left[ye^y - e^y - \frac{y^2}{2} \right]_{y=1}^2 = e^2 - \frac{3}{2}$$

Cvičení 3. Spočítejte $\iint_B \frac{y}{x^2} dx dy$, kde $B = \{(x, y) \in \mathbb{R}^2 : 1 \leq x \leq y \leq 3\}$.

Obor integrace:

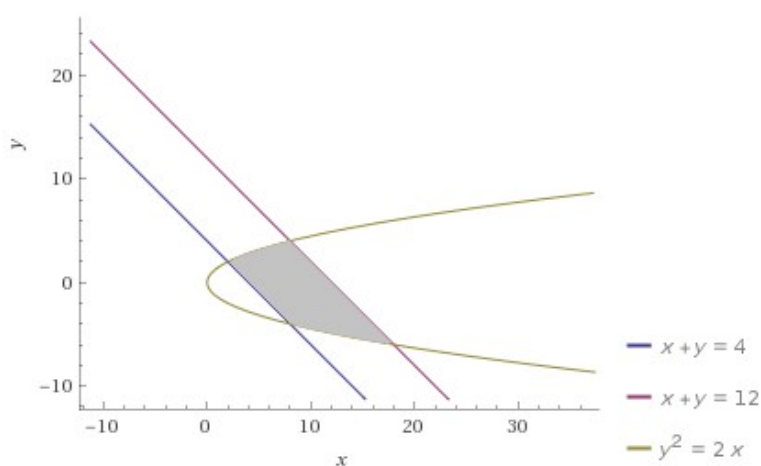


Řešení:

$$\iint_B \frac{y}{x^2} dx dy = \int_1^3 \int_x^3 \frac{y}{x^2} dx dy = \int_1^3 \frac{1}{x^2} \left[\frac{y^2}{2} \right]_{y=x}^3 dx = \int_1^3 \frac{9}{2x^2} - \frac{1}{2} dx = 2$$

Cvičení 4. Spočítejte $\iint_B 1 dx dy$, kde $B = \{(x, y) \in \mathbb{R}^2 : x + y \geq 4, x + y \leq 12, y^2 \leq 2x\}$.

Obor integrace:



Řešení:

$$\begin{aligned} \iint_B 1 dx dy &= \int_2^8 \int_{4-x}^{\sqrt{2x}} 1 dy dx + \int_8^{18} \int_{-\sqrt{2x}}^{12-x} 1 dy dx = \int_2^8 \sqrt{2x} - 4 + x dx + \int_8^{18} 12 - x + \sqrt{2x} dx \\ &= \frac{74}{3} + \frac{122}{3} \end{aligned}$$